Some Phenomena of Magnetic Disturbances at Kew

Charles Chree

Phil. Trans. R. Soc. Lond. A 1911 **210**, 271-306
doi: 10.1098/rsta.1911.0008

VIII. *Some Phenomena of Magnetic Disturbances at Kew.*

By Charles Chree, *Sc.D., LL.D., F.R.S., Superintendent of the Observatory Department of the National Physical Laboratory.*

(*From the National Physical Laboratory.*)

Received January 5,—Read February 10, 1910.

Contents.

Introductory.

§ 1. In the course of the present paper frequent reference is required to two previous papers, termed for brevity (A)* and (C),† which dealt with results derived from the Kew magnetograms of the eleven years 1890 to 1900.

(A) dealt with results from the Astronomer Royal's "quiet" days only, but included Declination, Inclination, Horizontal Force, and Vertical Force—termed sometimes for brevity D, I, H, and V. (C) dealt with declination only, but included results from all ordinary days and from the more highly disturbed days treated separately. Some results which had to be omitted from (C), or which are only glanced at there, were discussed in an appendix—termed here (C′)—attached to the reprint of (C) in Vol. V. of the 'Collected Researches' of the National Physical Laboratory.

The present paper is devoted to the phenomena on highly disturbed days, but

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* 'Phil. Trans.,' A, vol. 202, p. 335.

† 'Phil. Trans.,' A, vol. 208, p. 205.

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includes H, V, and I, and so, when taken in conjunction with (C) and (C′), affords a complete treatment of disturbance so far as it goes.

The disturbed days dealt with in (C)—see (C′) § 1A—numbered 209.

In selecting them regard was paid exclusively to the greater or less irregularity of the movement, and not to its mere amplitude. In dealing with magnetically quiet days at Kew, the practice has been to smooth the curves with a pencil so as to replace small oscillations or irregularities by curves of continuous curvature following the general trend of the undisturbed curve. This procedure was also followed in dealing with ordinary day curves. The 209 highly disturbed days were really those on which the declination curves were in part, or in whole, so irregular that it did not seem possible to smooth them satisfactorily, and which had thus to be omitted from the first essay at dealing with the regular diurnal inequality. There was no intention originally of utilising them in any way for the deduction of a diurnal inequality.

As was explained in (C), the curves for different years were under consideration at different times, so that no very uniform standard of disturbance probably existed. A further important consideration is that as declination was the only element under review in (C), declination curves only were consulted when the day was classified as disturbed.

During a magnetic disturbance one element is often much more disturbed than another, and the times at which the disturbance appears greatest in the different elements are usually different. Of the three elements D, H, and V, the last is unquestionably that which usually is least disturbed, while H is most. Thus, of the 209 days classed in (C) as disturbed, there are a number which would probably have been treated as ordinary if V had been the element under review, and there are even a fair number whose H curves might probably have been smoothed. On the other hand, there are a considerable number of days not included in the 209 which would certainly have been classed as disturbed if H had been the element originally dealt with. I decided, however, that arbitrary as the choice of the 209 days unquestionably was, from the point of view of the H and V curves, it was best to adhere to it so as to have all the disturbance results relating to a common set of days.

On several days the Kew trace was defective or beyond the limits of registration once or oftener during the 24 hours. Thanks to the kindness of Mr. E. KITTO, Superintendent of the Falmouth Observatory, it was possible to fill up nearly all these blanks fairly satisfactorily by reference to the Falmouth curves. On a few occasions the limits of registration at Kew and Falmouth were exceeded at one common hour during the 24. For these a reading was accepted answering to the edge of the photographic sheet. The error so introduced is believed to have been so small as to be practically without effect on the inequalities presently considered. On one occasion, however, August 20, 1894, the limits of registration in the H sheets at Kew and Falmouth were exceeded at several common hours, and it appeared best to omit the day when dealing with H. On the few days of incomplete trace the *absolute* range—*i.e.*, the excess of the absolute maximum over the absolute minimum during the 24 hours—was necessarily under-estimated.

Non-Cyclic Changes.

§ 2. The principal object of the present paper is to consider the diurnal inequalities derived from the disturbed days. The interpretation, however, of the phenomena, especially in the case of H, depends so considerably on the treatment of the non-cyclic changes, and these changes themselves are of so remarkable a character that their discussion calls for some detail. A general discussion of the non-cyclic changes at Kew on quiet days from 1890 to 1900 is given in (A), §§ 13 to 17. The mean values were for D $+0'\cdot044$ ($\equiv 0\cdot23\gamma$), for H $+3\cdot34\gamma$, for V $-0\cdot84\gamma$, and for I $-0'\cdot245$.

When first describing the phenomenon, I remarked* that the non-cyclic change in quiet days was "not unlikely to be only another phase of phenomena observed many years ago by General SABINE and Dr. LLOYD." These gentlemen, dealing respectively with Kew and Dublin results, described a marked tendency for disturbance to cause diminution in H, while SABINE observed a less marked tendency for disturbance to bring about increase of V. The diminution in the value of H during a magnetic storm is sometimes so large as to at once catch the eye. Dr. VAN BEMMELEN,† who discovered the phenomenon independently, regarded the non-cyclic effect apparently as simply a reaction or recovery from the effects of a storm.

When dealing in (C′) with non-cyclic changes of declination I found, somewhat to my surprise, that the mean value derived from the 209 disturbed days, viz., $+0'\cdot327$, had the same sign as the mean for the average quiet day, and that it was the "ordinary" days which supplied the requisite balance of negative non-cyclic changes. Another unexpected phenomenon was a peculiar oscillation of sign in non-cyclic changes. Out of 46 cases in which two, and only two, successive days were classified as disturbed, there were no less than 29 in which westerly declination diminished on the first day and rose on the second, whereas this particular sequence would have been expected on only 11 or 12 of the occasions. These two phenomena alone would suffice to show that the relation between the non-cyclic changes observed during quiet days and the alterations accompanying magnetic disturbance is of a somewhat complex character. One serious difficulty in the study of the connection is that magnetic disturbance is in reality the rule rather than the exception. An unprejudiced idea of the prevalence of disturbance will perhaps be best derived by reference to some data from the international quarterly lists published of late years in the Netherlands. Each day is classified at each contributory observatory as "0" (quiet), "1" (moderately disturbed), "2" (highly disturbed). It has not yet proved possible to secure a uniform standard at all the observatories, but at Greenwich, I believe, "0" includes the days

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* 'Brit. Assoc. Report' for 1895, p. 212.

† 'Met. Zeit.,' vol. 12, 1895, p. 321.

which are *quiet* according to the Astronomer Royal's standard, while "2" is limited to days which are so highly disturbed as to merit inclusion in the list of those the curves for which are reproduced in the annual Greenwich volume. The lists for Greenwich and Kew are got out independently. The figures suggest that the standard has varied at one at least of the two observatories, but the following results from the three most recent complete years give totals in truly remarkable agreement:—

	Greenwich.			Kew.		
Disturbance standard . . .	0	1	2	0	1	2.
Year 1906	139	217	9	166	194	5
„ 1907	117	237	11	118	236	11
„ 1908	170	187	9	143	210	13
Total of days . . .	426	641	29	427	640	29

The three years were in no way specially disturbed, rather the contrary, and yet over 60 per cent. of the days were classified as days of greater or less disturbance.

No day, it is true, is classified as "0" if any part of it is highly disturbed, while many days classified as "1" were doubtless of standard "0" during part of the 24 hours. On the other hand, quiet and disturbed days have rather a tendency to occur in groups, so that a disturbed day is more likely to have its adjacent days disturbed than the above figures would suggest. This is a feature which the international lists bring out in a striking fashion. During the three years the number of days classed as "2" was 29 at both Kew and Greenwich. Each list included 19 days common to the other list and 10 special to itself. The days immediately preceding and succeeding these days were classified as follows:—

	Days preceding—			Days following—		
Type	0	1	2	0	1	2.
Greenwich	4	16	9	0	20	9
Kew	6	16	7	0	22	7

Thus in no single case was a highly disturbed day followed by a really quiet one, and the number of instances in which it was preceded by a quiet day is much less than would have been anticipated from a consideration of the relative frequency of quiet and disturbed days.

The amount of disturbance during the 209 disturbed days of 1890 to 1900 was probably somewhat below the average for the 39 days classed as "2" at either Kew or Greenwich during the three years 1906 to 1908, but the difference was not great, and if data similar to those for the last three years had been in existence for 1890 to 1900, they would undoubtedly have disclosed a similar preponderance of disturbed days.

If each magnetic storm lasted a day, commencing at, say, 0h. 0m. a.m. and terminating exactly 24 hours later, and if the days immediately preceding were invariably free from oscillatory movements, one might, with some degree of assurance, associate with a storm the non-cyclic change observed subsequently. But even then there would be the possibility that during quiet days the earth's magnetism tended to rise above or fall below a stable value, and that the magnetic storm simply played the part of a mechanical shock, whose effects depended partly on the length of the preceding quiet interval. These remarks will, I hope, explain the way in which the non-cyclic results are presented.

§ 3. Table I. shows the mean non-cyclic increments, the 209 days being distributed amongst the months of the year to which they belong. The first line gives the number of disturbed days in each month. Results for D, already published, are included, but are measured in units of force instead of in angular measure as previously. Taking for H the mean value at Kew between 1890 and 1900, an increase of 1′ in westerly declination means a force of 5·32γ—where 1γ ≡ ·00001 C.G.S.—directed to magnetic west. The D, H, and V results are thus all expressed in terms of 1γ as unit; they are derived directly from the curve measurements. The data for I are calculated from the corresponding data for H and V; they are expressed, however, in angular measure. The results for H and I under August include August 20, 1904. If that day is excluded, the August values become for H −24·0γ and for I +1′·87; these are the values which had actually to be employed when dealing with the diurnal inequalities.

TABLE I.—Mean Values of the Non-cyclic Daily Changes.

	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
Number of days . .	22	24	30	17	16	6	10	10	15	25	20	14
Unit 1γ { D . . .	+ 2·9	− 4·6	− 1·1	− 6·8	− 2·1	− 1·2	+ 7·4	+16·1	+ 8·4	+ 4·9	+ 4·6	+ 1·6
H . . .	+ 0·1	−16·2	−22·1	−10·4	−10·0	−18·7	−26·3	−31·7	−18·1	− 4·9	−10·4	− 5·2
V . . .	+ 1·4	− 2·0	− 0·1	− 8·6	+ 5·0	+25·1	− 6·7	+ 9·5	+10·8	+ 2·0	+ 8·9	+ 6·2
I	+ 0′·03	+ 1′·02	+ 1′·47	+ 0′·46	+ 0′·81	+ 1′·94	+ 1′·57	+ 2′·38	+ 1′·51	+ 0′·38	+ 0′·94	+ 0′·52

§ 4. The number of disturbed days per month varied widely. If we allow equal weight to individual days, we get the means shown in Table II. This table also states the number of days during which the non-cyclic change was plus, zero, or minus.

TABLE II.—Non-cyclic Daily Change from all Disturbed Days.

	D.	H.	V.
Mean non-cyclic change . .	+ 1·74γ	− 13·2γ	+ 2·7γ
Number of days plus . . .	109	81	113
" " zero . . .	0	0	3
" " minus . .	100	128	93

The mean value from all disturbed days for I was +0′·96.

In the case of D, as already stated, the mean non-cyclic change from the 209 disturbed days has the same sign as that from the quiet days; in the case of H and V the signs for quiet and disturbed days differ. During one average disturbed day H fell as much as it rose on four average quiet days, while V rose as much as it fell on three average quiet days. It will be seen from Table I. that I is the only element in which the monthly means have all the same sign, though in H there is only one exception to the general rule. In all the elements the irregularities in the monthly means seem clear evidence that a much larger number of days would be required to give a reliable indication of seasonal variation.

§ 5. The monthly mean values already given are all that one requires for the elimination of the non-cyclic effect from the diurnal inequality, if we may assume it to proceed uniformly throughout the 24 hours. This is an assumption which cannot well be avoided in the present state of knowledge, but which, it is well to remember, may not be exact.

A clearer idea of the significance of the non-cyclic change may, however, be derived from Tables III. and IV. What suggested these tables was the declination phenomenon already described as occurring on a two-day disturbance. Of the 209 disturbed days 64 were preceded and followed by days which were not classified as disturbed. The remaining 145 days were made up of 46 sequences of two disturbed days, 9 sequences of three days, 5 sequences of four days, and 1 sequence of six disturbed days.

Table III. shows the mean non-cyclic changes when groups are formed containing all the days of isolated disturbances, all the "first" days when there was a sequence of disturbed days, and so on. Table IV. gives results for the same groups, only it splits each group into two, according as the non-cyclic change was plus or minus. The fifth and sixth days of the 6-day sequence are omitted, all the other disturbed days are included.

TABLE III.—Number of Days of several Groups and corresponding Mean Non-cyclic Changes.

	Isolated days.		First days.		Second days.		Third days.		Fourth days.	
	Number.	Mean non-cyclic change.	Number.	Mean non-cyclic change.	Number.	Mean non-cyclic change.	Number.	Mean non-cyclic change.	Number.	Mean non-cyclic change.
Declination	64	+ 5·1γ	61	−24·1γ	61	+22·3γ	15	+ 8·6γ	6	+6·7γ
Horizontal force . . .	64	−24·4γ	61	−40·4γ	61	+18·3γ	15	+13·8γ	6	−4·6γ
Vertical force	64	+ 2·9γ	61	−21·3γ	61	+27·1γ	15	+ 6·2γ	6	−5·5γ

TABLE IV.—Number of Days of + and − Non-cyclic Changes and corresponding Mean Values.

	Isolated days.				First days.				Second days.				Third days.				Fourth days.			
	−.		+.		−.		+.		−.		+.		−.		+.		−.		+.	
	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.	Number.	Mean.
Declination	27	27·8γ	37	29·1γ	42	49·6γ	19	32·2γ	19	26·7γ	42	44·5γ	7	22·9γ	8	30·5γ	3	13·7γ	3	27·0γ
Horizontal force . .	50	37·7γ	14	23·0γ	49	52·7γ	12	9·9γ	20	27·3γ	41	40·5γ	4	7·4γ	11	21·8γ	4	15·1γ	2	11·3γ
Vertical force* . . .	29	15·7γ	33	19·5γ	38	45·2γ	23	18·2γ	17	21·6γ	43	46·0γ	5	14·8γ	10	16·7γ	3	16·3γ	3	5·3γ

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* The non-cyclic change in Vertical Force was *nil* on 1 "second" and 2 "isolated" days.

§ 6. If we compare the results from the first and second days of a sequence of disturbed days, we find a truly remarkable difference. There is an unmistakable tendency to an oscillation, represented by a fall in all three elements during the first day and a rise during the second. There are, it is true, a considerable minority of occasions when the change was a rise on the first day and a fall on the second, and a very few when the change on both days was in the same direction, but the amplitude of the changes on such occasions was usually below the mean, especially in H. The number of "fourth" days was so small that the results are not of much significance. "Third" days in the case of H and V resemble "second" days, but the non-cyclic changes appear smaller.

The isolated days seem intermediate in character between "first" and "second" days, resembling the former most closely in the case of H, but the latter in the case of D and V. The isolated days represented on the whole the shortest storms, and if the phases vary in length with the length of the storm, one day in a short storm may suffice for the development of phenomena that require two days in a longer storm. It must, however, be remembered that it often depends on the hour of commencement whether highly disturbed conditions extend over one day or over two, also that the largest movements occur sometimes near the beginning, sometimes near the end, of a storm, and in the latter case the day on which the storm commences will very likely not be classed as highly disturbed. Another important point is that on individual days much turns on the "accident" of whether midnight comes at the crest or the hollow of the oscillatory movement in progress at the time. In a station at some distance east or west of Greenwich, *e.g.*, Falmouth, the sign even of the non-cyclic change would sometimes be altered if local time were substituted for G.M.T.

The relationship between the phenomena on the first and second days of a long storm can hardly be a simple case of action and reaction. The rise in V on the average "second" day distinctly exceeds the fall on the average "first" day. At the end of the disturbed period H on the average is still depressed, but D and V are slightly enhanced. The depression in H sometimes continues for a number of days or even weeks.

Another consideration is that the fall in H during "first" days of disturbance is often not the first change of the kind associated with the storm. When the storm has a sudden commencement, the sequence of changes in H at Kew is usually as follows:—In a minority of cases there is a distinct but minute fall lasting only one or two minutes, followed immediately by a considerably larger rise. In other cases no preliminary fall can be made out. In either event, in practically all cases, at the end of 5 minutes from the commencement H is above its initial value. There is a rapid rise usually of the order of 20γ, but sometimes 50γ, 70γ, or even 100γ. The rise may occupy only 2 or 3 minutes, but usually longer, and the time during which the force remains enhanced may be only a few minutes or over an hour.

Dr. VAN BEMMELEN,[*] who has made a special examination of the phenomena at a number of stations, found in most cases at least a trace of an oscillatory movement at the start, but the position at the end of 5 or 10 minutes—as pointed out long ago by W. ELLIS[†]—is usually a marked rise in H, as at Kew. At the Winter Quarters of the "Discovery,"[‡] in the Antarctic, the commencing movement appeared usually to be highly oscillatory, both movements being large, but the first of the two was sometimes a rise, sometimes a fall. These commencing movements are experienced, apparently simultaneously, all over the world, and for their elucidation we must look to the inter-comparison of data from a variety of stations. They are referred to here because a physical connection between them and the non-cyclic phenomena is not improbable. There is also occasionally a very direct but "accidental" connection, which may best be illustrated by reference to two individual cases. On November 4, 1892, a sudden commencing movement took place between 2 and 3 a.m., the immediate consequence of which was a rise of over 30γ in H in the course of a few minutes. The non-cyclic change on this day had first to wipe out this rise, but notwithstanding it left the value of H at the second midnight lower by 63γ than at the first midnight. On the other hand, on March 12, 1892, when the non-cyclic change during the 24 hours was a fall of no less than 209γ, the value at the first midnight was enhanced by at least 95γ above the normal owing to the sudden commencing movement, which took place between 10 and 11 p.m. on the 11th. March 12, 1892, further illustrates in a remarkable way the important part that may be played by the mere accident of time. The enhancement of H, following the sudden commencement of this storm, continued only for a few minutes after 0 a.m., and was then followed by a rapid fall. If the non-cyclic change had been taken for the 24 hours beginning at 0·15 a.m., it would not have amounted to a quarter of that actually recorded.

Diurnal Variation. Absolute Ranges.

§ 7. The usual aim when deriving diurnal inequalities is to obtain results representative of average normal conditions. In the case of Terrestrial Magnetism there is the complication that the amplitude of the regular diurnal inequality alters markedly with sunspot frequency. Highly disturbed days present, however, more outstanding sources of uncertainty. Let us suppose for a moment—what is not really true—that the value of, say, H at a particular hour is on disturbed days purely "accidental" so far as local time is considered. The diurnal inequality resulting from an infinite number of disturbed days would then vanish. But for a finite number of days a pseudo-inequality would naturally present itself, and its range would tend to increase as the number of disturbed days was reduced. If we came down to a single highly

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[*] 'K. Akademie van Wetenschappen te Amsterdam,' 1908, p. 773.

[†] 'R.A.S. Notices,' vol. 65, p. 520.

[‡] National Antarctic Expedition, 1901–1904, "Magnetic Observations," p. 179.

disturbed day we should obviously have an inequality of a kind with a very large range.

In the present instance mean monthly inequalities are based on a comparatively small number of days, in no case more than 30 and in one case only six.

There is thus a risk in all the months of a considerable pseudo-element in the apparent diurnal inequality, and the risk increases when the days of a particular month are specially few but include one or two of outstandingly large absolute range. Another possibility that requires to be taken into account is that a particular month of the year may accidentally contain an undue proportion of the most highly disturbed days—*i.e.*, a proportion much in excess of what it would have contained if we had been dealing with 100 years instead of 11—and consequently differences may appear between different months which are not really due to true seasonal variation.

Table V. is intended to elucidate the real character of the material utilised. It includes data for D as well as for H and V, expressing all in terms of force so as to facilitate comparison.

§ 8. The first column of Table V. gives the mean absolute daily range of D—in terms of 1γ as unit—derived from the quiet days of the 11 years, D being the only element for which these ranges have as yet been got out. The second column gives the corresponding mean values from the 209 disturbed days. The next two columns give the largest and least of the daily absolute ranges from the disturbed days. Of the two sets of results for H the first is also derived from the 209 days; the second gives for the 4 winter and 4 equinoctial months the results derived from the ten days of largest H range in each month. The maxima of range in this case are, of course, the same as for the whole 209 days, and so are not repeated. The first two sets of results for V are exactly analogous to those for H; the last set was obtained from the years 1892 to 1895 only, these being the years of largest sunspot frequency. In this case seasons only were considered, the means attached to, say, winter allowing equal weight to all individual days of the 4 months November, December, January, and February. Except in this case, seasonal means allow equal weight to the individual months, so that individual days are not equally potent.

One reason for deriving data from 10 days a month for the winter and equinoctial months, and for these only, was that only 42 of the whole 209 days fell in the four summer months May to August. There was thus room for suspicion that the average summer day might represent more highly disturbed conditions than the average winter or equinoctial day. The regular diurnal inequality is much reduced in amplitude in winter and so more easily obscured by disturbance, and thus a lesser degree of disturbance might suffice to get a winter day classified amongst the disturbed.

The quiet day D results in Table V. show a nearly constant range from April to August, a small but decided reduction in September and March, with a sharp fall to a minimum in December, and a more gradual recovery in January and February.

TABLE V.—Absolute Daily Ranges (Unit 1γ).

	Declination.				Horizontal force.					Vertical force.							
	Quiet day mean.	All disturbed days.			All disturbed days.			10 days a month.		All disturbed days.			10 days a month.		Sunspot maximum years.		
		Mean.	Greatest.	Least.	Mean.	Greatest.	Least.	Mean.	Least.	Mean.	Greatest.	Least.	Mean.	Least.	Mean.	Greatest.	Least.
January	33	141	262	80	105	275	45	137	97	52	162	11	75	44			
February	40	194	420	107	193	720	59	319	143	145	591	37	262	100			
March	56	181	455	55	159	493	46	268	147	135	566	30	269	115			
April	63	157	309	86	147	325	87	179	120	103	308	50	131	80			
May	64	194	412	114	183	306	89	—	—	171	407	35	—	—			
June	64	188	219	157	188	284	111	—	—	167	314	76	—	—			
July	62	204	410	103	231	650	85	—	—	192	638	44	—	—			
August	63	182	443	96	174	620	100	—	—	156	508	28	—	—			
September	58	180	307	90	176	495	69	213	136	137	356	39	175	90			
October	49	138	192	72	103	191	61	135	100	59	140	16	85	61			
November	35	177	287	110	134	348	48	167	130	105	429	22	149	90			
December	27	155	268	64	114	219	62	130	87	77	280	2	100	38			
Year	51	174			159			—		125			—		178	638	28
Winter	34	167			137			188		95			147		153	591	35
Equinox	56	164			146			199		109			165		153	362	28
Summer	63	192			194			—		172			—		228	638	28

The corresponding disturbed days' data present irregularities which are presumably due to the paucity of days. The figures strongly suggest that during the 11 years October and April were less disturbed, but November more disturbed than usual; February seems also a little outstanding as compared to March.

The 11 years contained few very large disturbances. The largest and smallest D ranges included in the list, 455γ (85$'$·6) and 55γ (10$'$·3), both belong to March. The former may have been surpassed on February 14, 1892, when the limit of registration was exceeded. The 10$'$·3 range was exceptional. On 5 of the 12 months no range under 100γ was included, and few ranges less than this occurred in the remaining 7 months.

The H and V figures in Table V. go beyond those for D in representing October and April as abnormally quiet, and December and January present very small ranges, especially in V, as compared to the summer months. Table V. certainly suggests that on disturbed days the seasonal variation in the absolute ranges is much larger in V and even in H than in D, but this wants confirmation from a more extended enquiry.

As regards the absolute size of the ranges in the three elements, the mean value for D is the largest in every month, except July, and that for V is invariably the least. The inference, however, that D is the most disturbed element would be incorrect. It must be remembered that the regular diurnal inequality is greater in D than in the other elements on undisturbed days. The ranges in the mean diurnal inequalities for the year from the quiet days of 1890 to 1900 were 42·0γ (7$'$·90) in D, and only 26·2γ in H, and 17·9γ in V. Thus, relative to the ranges of the regular diurnal inequality, the absolute ranges for H and V are not less but larger than the ranges in D. Further, Table V. is derived from days in which the criterion of disturbance was the character of the D curve alone. A good many days were included amongst the 209 which would have been excluded if the criterion had been disturbance in H or in V.

A remarkable example of this is afforded by December 21, 1897. The range in D was 103γ (19$'$·4), and that in H 86γ, but the V trace to the eye appeared practically a straight line. Under such circumstances one is very apt to miss the absolute maximum and minimum, and the true absolute range may have slightly exceeded the value 2γ found by measurement. But, for all practical purposes, the inclusion of this day amongst the disturbed simply tended to reduce the inequality range in V by some 7 per cent. This was an extreme case—the next smallest absolute range for V being 11γ; still amongst the 209 days there were an appreciable number, especially in the winter months, which, so far as V was concerned, were essentially quiet days.

Comparing the results from the whole number of disturbed days with those from the 10 days a month of largest range, we see that the uniformity secured in the latter case in the number of days, whilst reducing the variability amongst the days of each month, increased on the whole the differences between the different months. The reduction in number was thus not without its disadvantages. By having, however, independent results from a smaller number of more highly disturbed days, one was able to arrive at a clearer idea of the relationship between the absolute range and the range of the diurnal inequality.

The object of obtaining results confined to the years of sunspot maximum was to see whether disturbed days showed any parallel to the great enhancement of the regular diurnal inequality on quiet and ordinary days at sunspot maximum. The four years 1892 to 1895 contributed 33 disturbed days to winter, 28 to equinox, and 20 to summer. The mean value of the absolute range for these days, it will be noticed, was considerably in excess of the mean from the whole 11 years, a fact that should be borne in mind when considering the corresponding diurnal inequalities.

TABLE VI.—Diurnal Inequality of Horizontal Force (Unit $0 \cdot 1\gamma$).

	Forenoon.											
Hour	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January	−17	−24	−40	+50	+82	**+106**	+97	+73	+20	−3	−44	−45
February	**−190**	−124	−34	−22	−6	+86	+62	+28	−58	−97	−134	−179
March	−65	−42	**+44**	**−3**	+23	+53	+25	−43	−123	**−172**	−165	−109
April	+130	+99	+75	+36	+119	+71	−33	−154	−312	**−335**	−305	−295
May	+92	+23	+124	−66	+11	−113	−135	−222	−331	**−447**	−413	−318
June	−103	+116	+68	+40	+51	−25	−82	−480	−684	**−696**	−477	−203
July	−29	+40	+8	+61	−222	−247	−435	−426	−432	−520	**−623**	−417
August	+92	+102	+74	+179	+21	−126	−351	−462	−482	**−518**	−425	−374
September	+66	+42	**+198**	−10	+148	+67	−27	−165	−330	**−333**	−298	−215
October	+65	+42	+96	+60	**+122**	+103	+78	−22	−165	**−232**	−203	−114
November	+42	+74	+76	**+195**	+155	+123	+116	+30	−70	−104	**−171**	−97
December	−53	−49	+7	+49	**+156**	+104	+138	+140	+56	+36	−33	+6

	Afternoon.												Range.	Average departure from the mean.
Hour . . .	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.		
January . . .	−64	−16	+41	−18	+11	−13	−19	−88	+65	−38	**−101**	−15	207	45
February . . .	+18	+73	**+191**	+98	+134	+54	+77	+98	+9	−134	+13	+37	381	81
March	+5	+54	+94	+148	**+150**	+145	+55	+18	−38	+32	−32	−54	322	71
April	−91	−65	+122	+115	+224	+172	**+231**	+87	+113	−58	+16	+38	566	137
May	−185	+15	+212	+334	+338	+325	**+347**	+101	+110	+87	+60	+51	794	186
June	−38	+60	+186	+231	+327	+313	+365	**+425**	+117	+253	+149	+87	1121	232
July	−261	+412	+186	+525	+512	+406	**+618**	+354	+268	−16	+322	−84	1241	309
August	−89	−21	+137	+251	+431	**+520**	+409	+314	+96	−28	+58	+192	1038	240
September . . .	−89	+101	+82	+99	+167	+177	+168	−13	−73	+81	+136	+21	531	129
October	−95	−100	−60	−30	−18	+42	+22	+71	+112	+92	+55	+79	354	87
November . . .	−141	−33	−83	−69	−21	−26	−6	−56	+26	+28	−16	+28	366	74
December . . .	−55	−37	−41	−88	−28	+76	−14	−16	−84	−71	**−147**	−52	303	64

Diurnal Inequalities from Disturbed Days.

§ 9. Tables VI. to IX. give the regular diurnal inequalities derived from the 209 disturbed days—the curves being measured unsmoothed at the exact hours as in (C)—after eliminating the non-cyclic change in the usual way. These non-cyclic changes were much smaller for V than for H, so that any uncertainty arising from the method of elimination is presumably much less for the former element than the latter. The same temperature corrections were applied as in the case of the quiet days in (A). To avoid decimals $0 \cdot 1\gamma$ has been taken as the unit, following the example afforded by (A). The algebraically largest and smallest hourly values are in heavy type; their difference is taken as the range of the inequality. By the

TABLE VII.—Diurnal Inequality of Vertical Force (Unit $0 \cdot 1\gamma$).

	Forenoon.											
Hour	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January	− 77	−102	**−106**	− 85	− 81	− 73	− 73	− 69	− 68	− 73	− 51	− 50
February	**−401**	−343	−308	−312	−243	−219	−187	−156	−127	−124	− 77	− 44
March	−156	**−235**	−222	−197	−197	−197	−140	−116	−116	−119	−123	− 94
April	−155	−148	−149	−189	−178	−174	−149	−150	−177	−189	**−199**	−157
May	−320	−327	−255	**−336**	−324	−272	−231	−210	−241	−248	−218	−143
June	−240	−291	**−297**	−218	−229	−265	−258	−292	−274	−269	−136	− 63
July	−198	−299	−426	−530	**−584**	−451	−332	−254	−203	−195	−184	−126
August	− 44	− 98	−170	−413	−540	**−577**	−365	−210	−139	− 82	− **4**	− 2
September	−148	−226	−208	**−302**	−200	−152	− 94	− 81	− 91	− 45	− 82	− 52
October	−108	−119	−119	**−121**	−104	− 98	− 77	− 77	− 79	− 99	−102	− 63
November	−224	**−245**	−241	−209	−184	−160	−139	−120	− 83	− 72	− 44	− 13
December	− 47	− 97	**−166**	−120	− 89	−110	− 86	− 77	− 78	− 96	− 87	− 72

	Afternoon.												Range.	Average departure from the mean.
Hour . . .	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.		
January. . . .	−17	+ 53	+ 83	+112	+116	+104	+127	**+129**	+105	+62	+ 44	− 10	235	78
February . . .	+89	+189	+339	+400	**+444**	+422	+395	+247	+178	+20	− 16	−166	845	227
March	− 8	+ 86	+194	+315	+392	**+447**	+404	+289	+166	+17	−184	−206	682	192
April.	−41	+ 72	+244	+296	+398	**+421**	+369	+256	+139	+25	− 23	−139	620	185
May	+65	+269	+504	**+648**	+610	+515	+398	+284	+150	−14	−115	−189	98	287
June.	+ 3	+142	+429	**+662**	+560	+518	+411	+276	+176	−52	− 86	−207	959	265
July	+23	+379	+360	+662	**+758**	+613	+509	+409	+283	+47	− 78	−184	1342	337
August	+68	+127	+254	+317	+428	**+520**	+420	+192	+225	+28	+ 68	− 3	1097	221
September. . .	+18	+120	+274	+361	+380	**+418**	+298	+123	− 23	−69	− 33	−186	720	166
October. . . .	−18	+ 65	+155	+202	**+213**	+188	+185	+133	+ 93	+30	− 12	− 68	334	105
November . . .	+64	+147	+270	**+384**	+325	+266	+244	+168	+ 42	+35	− 54	−157	629	162
December . . .	−33	+ 20	+129	**+216**	+162	+156	+130	+120	+ 83	+81	+ 65	− 4	382	97

"average departure from the mean" is meant the quotient when the arithmetical sum of the 24 hourly differences from the mean for the day is divided by 24.

Tables VI. and VII. contain the H and V inequalities for individual months of the year based on all the disturbed days. The V inequalities are plotted in the curves of fig. 1. These curves, it is hoped, will carry conviction of the fact that the main features of the monthly inequalities are not accidental.

Fig. 1. – *Vertical Force.*

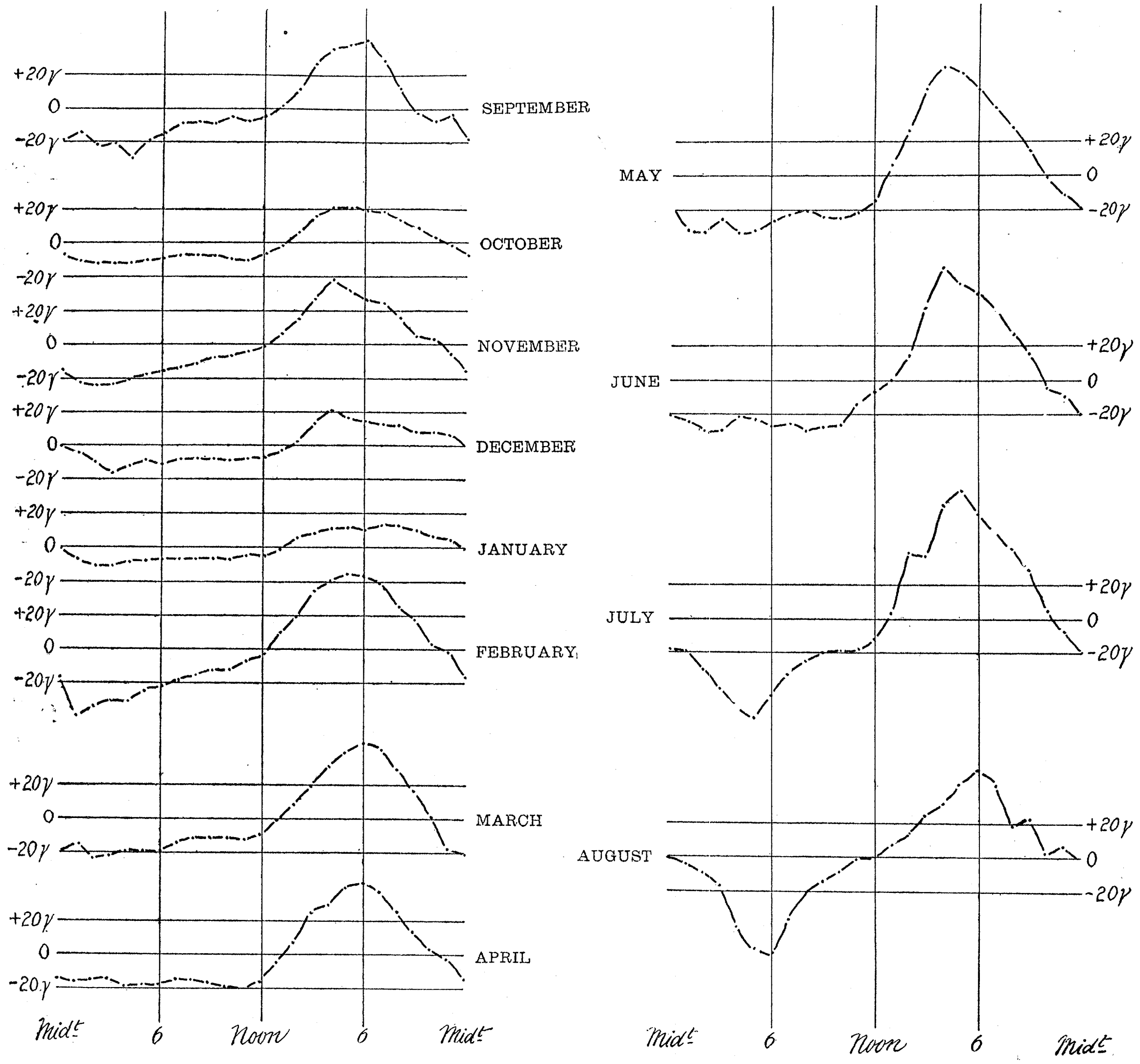


Table VIII. gives the H and V diurnal inequalities for the year and the four seasons, based first on all the disturbed days, then for winter and equinox on the 10 days a month of largest range, and, finally, in the case of V, on the disturbed days of the four years 1892 to 1895 only. In forming the seasonal variations equal weight was allowed to each month, irrespective of the number of days, except in the case of the

TABLE VIII.—Diurnal Inequalities for the Year and Three Seasons (Unit 0·1γ).

	Hour	Forenoon.											
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
H	Year, all days	+ 2	+ 25	+ 58	+ 47	+ 55	+ 17	− 46	−142	−243	**−285**	−274	−197
	Winter, all days	− 54	− 31	+ 2	+ 63	+ 97	**+105**	+103	+ 68	− 13	− 42	− **95**	− 79
	,, 40 days	− 96	− 99	− 26	+ 63	+ 88	+ 96	+ **97**	+ 57	− 8	− 34	− 92	− 90
	Equinox, all days.	+ 49	+ 35	+103	+ 21	+103	+ 73	+ 11	− 96	−233	**−268**	−243	−183
	,, 40 days	+ 34	− 2	+120	+ 6	+108	+ 72	− 17	−143	−272	**−312**	−266	−192
	Summer, all days.	+ 13	+ 70	+ 68	+ 54	− 35	−128	−251	−398	−482	**−545**	−484	−328
V	Year, all days . . , ...	−176	−211	−222	**−253**	−246	−229	−178	−151	−140	−134	−109	− 73
	,, sunspot maximum	−295	−321	−334	**−387**	−367	−330	−241	−182	−157	−132	− 92	− 41
	Winter, all days	−187	−197	**−205**	−182	−149	−141	−121	−106	− 89	− 91	− 65	− 45
	,, 40 days	−334	**−339**	−336	−282	−213	−190	−153	−120	− 92	− 97	− 61	− **47**
	,, sunspot maximum . . .	−401	**−402**	−377	−327	−248	−201	−155	−123	− 93	− 89	− 50	− 24
	Equinox, all days.	−142	−182	−175	**−202**	−170	−155	−115	−106	−116	−113	−126	− 92
	,, 40 days	−225	−277	−258	**−284**	−239	−221	−164	−143	−135	−120	−122	− 66
	,, sunspot maximum . .	−244	−266	−266	**−300**	−241	−232	−165	−130	−123	−131	−132	− 70
	Summer, all days.	−200	−254	−287	−374	**−419**	−391	−296	−242	−214	−198	−135	− 83
	,, sunspot maximum . .	−239	−296	−358	−533	**−613**	−557	−402	−292	−255	−177	− 95	− 30

	Hour	Afternoon.												Range.	Average departure from the mean.
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.		
H	Year, all days	− 90	+ 37	+ 89	+133	+186	+183	**+188**	+108	+ 60	+ 19	+ 43	+ 27	473	106
	Winter, all days	− 61	− 3	+ 27	− 19	+ 24	+ 23	+ 9	− 15	+ 4	− 54	− 63	− 1	200	44
	,, 40 days.	− 31	+ 44	+ 76	+ 42	+ 87	+ 50	+ 53	− 27	− 6	**−137**	−107	− 2	234	63
	Equinox, all days	− 68	− 2	+ 59	+ 83	+131	**+134**	+119	+ 41	+ 29	+ 37	+ 44	+ 21	402	91
	,, 40 days	− 46	+ 45	+123	+128	+158	**+173**	+150	+ 46	+ 41	+ 10	+ 44	− 8	485	105
	Summer, all days	−143	+116	+180	+335	+402	+391	**+435**	+299	+148	+ 74	+147	+ 62	980	233
V	Year, all days	+ 18	+139	+270	+381	**+399**	+382	+324	+219	+135	+ 17	− 35	−127	652	190
	,, sunspot maximum .	+ 73	+213	+368	+516	**+540**	+534	+452	+289	+184	+ 9	− 80	−218	927	265
	Winter, all days	+ 26	+102	+205	**+278**	+262	+237	+224	+166	+102	+ 50	+ 10	− 84	483	138
	,, 40 days.	+ 59	+156	+310	**+426**	+378	+342	+319	+229	+133	+ 52	+ 12	−152	765	201
	,, sunspot maximum	+ 81	+170	+324	**+423**	+419	+386	+388	+257	+148	+ 60	+ 4	−170	825	222
	Equinox, all days	− 12	+ 86	+217	+293	+346	**+368**	+314	+200	+ 94	+ 1	− 63	−150	570	160
	,, 40 days	+ 31	+162	+325	+418	+479	**+497**	+414	+256	+109	− 25	−151	−261	781	224
	,, sunspot maximum	+ 48	+152	+276	+374	+435	**+486**	+437	+287	+160	+ 32	−117	−270	786	224
	Summer, all days	+ 40	+229	+387	+572	**+589**	+541	+434	+290	+208	+ 2	− 53	−146	1008	274
	,, sunspot maximum	+ 90	+318	+503	+752	**+765**	+731	+530	+322	+244	− 65	−128	−215	1378	355

results from sunspot maximum years for V, when equal weight was allotted to the individual days.

§ 10. Table IX. gives mean diurnal inequalities for the year and the three seasons for the northerly component N, the westerly component W, and the inclination I, based on the whole 209 days.

TABLE IX.—Diurnal Inequalities of Northerly and Westerly Components (Unit 0·1γ) and of Inclination.

Element.		Forenoon.											
	Hour	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
N . .	Year	+78	+88	+112	+76	+60	+24	−38	−123	−231	−299	**−322**	−285
	Winter	+14	+22	+42	+81	+86	+82	+74	+48	−30	−73	−139	**−147**
	Equinox	+108	+93	+157	+55	+122	+86	+27	−77	−227	−295	**−313**	−292
	Summer	+116	+146	+135	+94	−29	−98	−212	−339	−436	**−529**	−514	−413
W . .	Year	**−246**	−197	−163	−86	−6	−19	−34	−83	−75	+2	+113	+252
	Winter	−227	−176	−127	−31	+50	+89	+110	+76	+52	+95	+128	+208
	Equinox	−181	−181	−158	−107	−45	−31	−48	−76	−56	+46	+187	+323
	Summer	**−328**	−235	−205	−120	−24	−115	−164	−250	−221	−136	+23	+224
		′	′	′	′	′	′	′	′	′	′	′	′
I . . .	Year	−0·51	−0·75	−1·01	−1·02	**−1·05**	−0·75	−0·19	+0·53	+1·23	**+1·53**	**+1·53**	+1·11
	Winter	−0·16	−0·34	−0·59	−0·96	−1·06	**−1·09**	−1·02	−0·74	−0·16	+0·02	+0·46	+0·40
	Equinox	−0·72	−0·74	**−1·17**	−0·70	−1·16	−0·92	−0·39	+0·34	+1·23	**+1·47**	+1·27	+0·97
	Summer	−0·65	−1·17	−1·26	−1·40	−0·93	−0·23	+0·85	+1·98	+2·62	**+3·08**	+2·85	+1·95

Element.		Afternoon.												Range.	Average departure from the mean.
	Hour . . .	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.		
N . .	Year	−197	−77	−6	+63	+136	+169	**+202**	+158	+128	+79	+118	+87	524	131
	Winter	−136	−86	−38	−69	−2	+31	+30	+18	**+94**	+21	+21	+56	241	60
	Equinox . . .	−202	−136	−45	+19	+85	+128	**+163**	+127	+105	+122	+113	+76	476	132
	Summer . . .	−255	−9	+63	+240	+323	+347	**+411**	+329	+186	+95	+220	+130	940	236
W. .	Year	+331	**+372**	+320	+244	+189	+73	−16	−143	−210	−191	−236	−190	618	158
	Winter	+233	**+266**	+212	+158	+86	−22	−66	−109	**−288**	−251	−281	−185	554	147
	Equinox . . .	+420	**+430**	+344	+219	+166	+40	−124	−269	−241	**−268**	−215	−175	698	181
	Summer . . .	+339	**+421**	+404	+355	+314	+200	+143	−52	−100	−55	−211	−209	749	202
		′	′	′	′	′	′	′	′	′	′	′	′	′	′
I . .	Year	+0·65	+0·14	+0·16	+0·17	−0·12	−0·16	−0·35	−0·11	−0·03	−0·08	−0·38	−0·54	2·58	0·59
	Winter	+0·47	+0·30	+0·39	**+0·90**	+0·57	+0·51	+0·56	+0·56	+0·26	+0·50	+0·45	−0·23	1·99	0·53
	Equinox . . .	+0·41	+0·25	+0·20	+0·26	+0·09	+0·13	+0·08	+0·28	+0·07	−0·24	−0·47	−0·56	2·64	0·59
	Summer . . .	+1·06	−0·14	−0·13	−0·64	−1·02	−1·10	**−1·69**	−1·18	−0·41	−0·49	−1·13	−0·82	4·77	1·20

These inequalities are not deduced directly from the curve measurements but are calculated from the corresponding inequalities in D, H, and V through the relations

$$\Delta N = \Delta H \cos D - H\Delta D \sin D,$$

$$\Delta W = \Delta H \sin D + H\Delta D \cos D,$$

$$\Delta I = \tfrac{1}{2} \sin 2I\,(\Delta V/V - \Delta H/H),$$

where Δ represents departure from the mean value for the day.

The numerical formulæ actually employed, accepting their mean values for D, H, and V, were

$$\Delta N = 0{\cdot}955\Delta H - 1{\cdot}58\Delta D,$$

$$\Delta W = 0{\cdot}298\Delta H + 5{\cdot}08\Delta D,$$

$$\Delta I = 0{\cdot}0278\Delta V - 0{\cdot}0667\Delta H.$$

In the numerical formulæ ΔD and ΔI are expressed in terms of $1'$ as unit, while 1γ is the unit for ΔH, ΔV, ΔN and ΔW.

§ 11. As already explained, there is reason to expect a not inconsiderable pseudo-element in the diurnal inequalities of individual months. When the number of days available is large, the most satisfactory way of distinguishing between the normal and the accidental is to compare results based on two or more independent sets of days belonging to the same month of the year. But the same object is usually achieved pretty satisfactorily by comparing results from different months of the same season. This second method, of course, will break down if the seasonal variation in type is very great, but that is rather an unlikely contingency in temperate latitudes.

The results for H in Table VI. present irregularities in the successive hourly values which are obviously of an "accidental" character, but the main features are clearly not "accidental." In summer there seems to be only a single maximum and minimum, the former about 7 p.m., the latter about 10 a.m. In equinox, and still more in winter, there is evidence of a double oscillation, and in December and January the forenoon maximum and the evening minimum are apparently the principal ones. A closely similar difference between summer and winter was described in (A) for the case of quiet days.

The range of the diurnal inequality in H from the disturbed days bears to that from quiet days a ratio varying from 1·26 : 1 to 3·24 : 1. The mean of the twelve ratios 2·08 is a little in excess of the corresponding mean 1·95 found in the case of D. But whereas the average of the monthly ratios for the three seasons was in the case of D for winter 2·79, equinox 1·62, and summer 1·43, the corresponding figures in the case of H are winter 2·08, equinox 1·42, and summer 2·74. The position occupied by summer is thus exactly reversed in the two cases. The same phenomenon appears, but in an even more striking way, in the seasonal results in Table IX. for N and W. In the case of quiet days the range in W bore to that in N a ratio varying from 1·25 in December (representing winter) to 1·58 in March and in June (respectively representing equinox and summer). But on disturbed days we see from the figures in Table IX. that this ratio has risen to 2·30 in winter and fallen to 0·80 in summer.

The seasonal variation in the shape of vector diagrams—the curves illustrating the diurnal changes in the horizontal components D and H or W and N—is thus totally different for quiet and disturbed days.

§ 12. Taking the year as a whole, the ratio borne by the diurnal inequality range in W to that in N is only 1·18 on disturbed days as compared to 1·46 on quiet days. This is due to the fact that the increase in the range of the mean diurnal inequality as we pass from quiet to disturbed days is 91 per cent. in N as against 54 per cent. in W. In this instance the change in passing from quiet to disturbed days is in the same direction as that observed on quiet days when we pass from years of few to years of many sunspots.

The mean diurnal inequality for the year in N is, on the whole, of the same type on disturbed and quiet days. The hour of principal maximum, 7 p.m., and principal minimum, 11 a.m., are the same in the two cases.

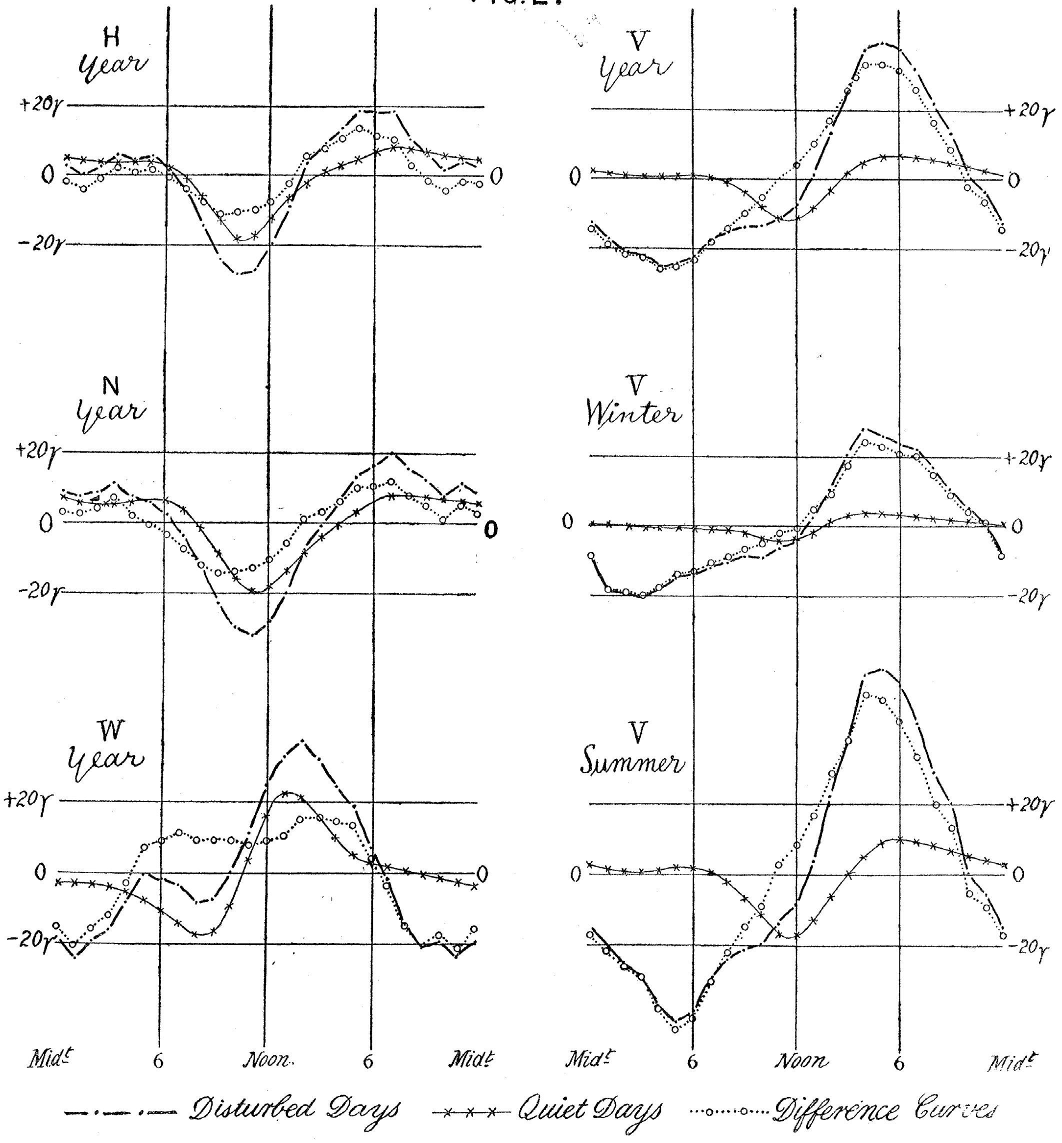
There is much more difference of type in the case of W. The principal minimum appears near midnight on disturbed days as compared with 8 a.m. on quiet days. The most conspicuous turning point, however, the maximum, occurs in both cases in the early afternoon, though somewhat later on the disturbed than on the quiet days. The nature of the differences between quiet and disturbed days can best be followed by reference to fig. 2, p. 290.

§ 13. The increase in diurnal inequality range on disturbed days in D and H, though considerable, is quite overshadowed by that in V. Comparing individual months in Table VII. with the corresponding monthly data on quiet days ((A), Table XIV., p. 363), we find that the range from disturbed days bears to that from quiet days a ratio whose least value is 2·20 in October, and whose mean values are 2·91 for equinoctial, 4·05 for summer, and 6·31 for winter months. The mean of the ratios from the twelve months, 4·42, is more than double the corresponding means for D and H. The size of the disturbance influence appeals to the eye in the V curves in fig. 2. These show the diurnal inequalities for the whole year and for winter and summer for both disturbed and quiet days, as well as an inequality derived by subtracting quiet day hourly values from disturbed day values. This last inequality may be regarded from one point of view as representing the disturbance element pure and simple.

It will be observed in Table VIII. that the excess in the inequality range, shown by the smaller number of more disturbed days, is much more conspicuous in V than in H. The sunspot maximum years show an even more enhanced inequality range in V, especially in winter.

§ 14. The type of the diurnal inequality on disturbed days in V, as shown in Tables VII. and VIII. and in figs. 1 and 2, does not vary much with the season, but it differs somewhat conspicuously from that observed on quiet days. The principal or only minimum for the day occurs on disturbed days in the early morning—at 4 a.m. in the inequality for the year—but on quiet days it occurs at 11 a.m. or noon. The difference is most readily grasped by reference to fig. 2.

FIG. 2.



Presumably there is a gradual transition in the type of the inequality in V as we pass from the quietest to the most disturbed of days. It will, in fact, be noticed in Table VIII. that the negative hourly values near 11 a.m. are numerically larger in the inequalities derived from all the days than in those derived from the more highly disturbed days, though earlier in the morning near the time of the minimum the latter are much the larger. This doubtless represents the influence which on days free from disturbance makes the minimum come at about 11 a.m.

§ 15. Diurnal inequalities in I were calculated for the individual months of the year, but Table IX. contains only the seasonal results. Even these leave something to be desired as regards smoothness. In the mean diurnal inequality for the year in I the principal maximum occurs at 10 a.m. as in the case of quiet days, but the principal minimum is found at 5 a.m., instead of 7 p.m. as on quiet days. The range in the mean inequality for the year bears to the corresponding range from quiet days the ratio 1·82 : 1, which is a little less than the corresponding values for D and H.

In all the elements disturbance is thus associated with a marked increase in the range of the diurnal inequality. The same phenomenon was described recently* at the "Discovery's" Winter Quarters in latitude 77° 51′ S., where the omission of 47 per cent. of the days, retaining only those of lesser disturbance, reduced the range of the mean diurnal inequality for the year in D by about 25 per cent. Some stations are much more disturbed than others, thus diurnal inequalities got out from any choice of days answer to different amounts of disturbance at different stations. The irregular movements presented by the average day's curves in high latitudes are such that their appearance in a Kew curve would inevitably lead to its inclusion amongst the highly disturbed. It is thus a question what is the real significance of the very large amplitude seen in the diurnal inequality at such a highly disturbed station as the "Discovery's" Winter Quarters. What would the amplitude of the inequality be in such a case if we had a large number of year's data at our disposal and retained only those for the quietest day of each month ?

§ 16. If all days were quiet and alike, and if the daily maximum and minimum happened to occur at exact hours G.M.T., then the inequality range would equal the absolute range. The interval of time between the daily maximum or minimum and the nearest hour cannot exceed 30 minutes. If the diurnal inequality could be represented by a simple wave of 24-hour period, 30 minutes of time would answer to a change of 7°·5 in the phase angle, and the largest hourly value could not be less than 0·99 of the true maximum.

Most diurnal inequalities are of a considerably more composite character, but sharp peaked forms are rare, thus non-coincidence in time of either maximum or minimum with an exact hour is unlikely to reduce the range derived from hourly readings by more than 3 or 4 per cent. Even on the quietest day, however, magnetic curves exhibit small irregularities, and the times of maximum and minimum vary from day to day. Thus the absolute range presents a limit which the inequality range may theoretically attain to, but which it is practically certain to fall considerably short of.

Passing from quiet to disturbed days, we may expect a large reduction in the ratio borne by the inequality range to the mean absolute range, and this for several reasons. On highly disturbed days sharp peaks are not uncommon, and the maximum and

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* National Antarctic Expedition, 1901–1904, "Magnetic Observations," p. 102.

minimum for the day thus often differ much from the largest and smallest hourly values. Then disturbed day phenomena are often in the highest degree irregular, and it is thus natural to expect a much greater variability in the times of maximum and minimum. The more variable these times, and the more variable the type of disturbance, the smaller will the ratio of the inequality range to the absolute range tend to be. It thus appeared desirable to calculate the values of the ratio for the disturbed days in the several elements, and to compare them with the only corresponding quiet and ordinary day data available, those for D. This comparison is effected in Table X. It contains two sets of values. The first set represents the arithmetic mean values of the ratio (inequality range/absolute range) calculated for the 12 months individually. The second set was derived from the seasonal diurnal inequalities and the corresponding mean absolute ranges.

TABLE X.—Values of (Inequality Range ÷ Absolute Range).

		Year.	Winter.	Equinox.	Summer.
Arithmetic means from months forming the season	D quiet days	0·809	0·669	0·861	0·898
	„ ordinary days	0·657	0·502	0·673	0·797
	„ all disturbed days	0·426	0·373	0·482	0·424
	H all disturbed days	0·361	0·233	0·308	0·541
	„ 10 disturbed days a month	—	0·244	0·335	—
	V all disturbed days	0·574	0·534	0·550	0·638
	„ 10 disturbed days a month	—	0·542	0·530	—
From seasonal diurnal inequalities	D quiet days	0·822	—	—	—
	„ all disturbed days	0·367	0·349	0·461	0·392
	H all disturbed days	0·298	0·147	0·275	0·505
	„ 10 disturbed days a month	—	0·122	0·244	—
	V all disturbed days	0·522	0·511	0·525	0·587
	„ 10 disturbed days a month	—	0·521	0·473	—
	„ disturbed days sunspot maximum	0·521	0·539	0·515	0·603

Difference in the type of a diurnal inequality between the months comprised in a season tends to reduce the inequality range for the season below the arithmetic mean of the ranges for the contributory months. Thus if the type of the diurnal inequality in individual months were largely dependent on "accidental" features of individual disturbances—in other words, if there were a large pseudo-element—the second set of values in Table X. for disturbed days would be much smaller than the first set. The second set, it will be seen, is the smaller in all cases, the most conspicuous example being the winter value for H; but in most cases, especially in V, the difference between the two sets of values is small.

As might have been anticipated, the largest ratio in Table X. is for the quiet day D results; and there is a substantial reduction in passing even to the ordinary days. The most remarkable feature of the table is the large size of the ratio in the case of V. The winter values for this element actually exceed the winter value for D on ordinary days. Another important result is the large size of the value for H in summer.

Comparing the results obtained for V from all disturbed days, from 10 days a month, and from sunspot maximum years, we see that the inequality range increases in all cases with the absolute range, and is roughly proportional to it.

§ 17. In comparing diurnal inequalities attention is often directed almost entirely to the amplitude of the range. This gives, however, rather a one-sided view. The value of a particular element may differ very little from the mean throughout the greater part of the 24 hours, the changes to and from the mean being practically all concentrated in a few hours. A second element may pass rapidly from a value near the maximum to a value near the minimum and conversely, retaining throughout the greater part of the 24 hours values remote from the mean. The curves representing

Table XI.—Values of (Average Departure ÷ Diurnal Inequality Range).

		Year.	Winter.	Equinox.	Summer.
Arithmetic means from months forming the season	D quiet days	0·221	0·241	0·208	0·213
	,, all disturbed days	0·254	0·252	0·260	0·251
	I quiet days	0·239	0·219	0·247	0·251
	,, all disturbed days	0·237	0·233	0·245	0·233
	H quiet days	0·235	0·204	0·248	0·254
	,, all disturbed days	0·227	0·212	0·238	0·230
	,, 10 disturbed days a month . .	—	0·210	0·227	—
	V quiet days	0·226	0·232	0·223	0·223
	,, all disturbed days	0·271	0·278	0·281	0·255
	,, 10 disturbed days a month . .	—	0·255	0·273	—
From seasonal diurnal inequalities	D quiet days	0·218	—	—	—
	,, all disturbed days	0·268	0·259	0·265	0·253
	I quiet days	0·238	—	—	—
	,, all disturbed days	0·228	0·266	0·223	0·251
	H quiet days	0·244	—	—	—
	,, all disturbed days	0·225	0·221	0·227	0·238
	,, 10 disturbed days a month . .	—	0·273	0·216	—
	V quiet days	0·221	—	—	—
	,, all disturbed days	0·292	0·287	0·281	0·272
	,, 10 disturbed days a month . .	—	0·263	0·287	—
	,, disturbed days sunspot maximum	0·285	0·269	0·285	0·257

the inequality would be very peaked in one case and very flat topped in the other. If the range was the same in the two inequalities, the curves would represent the action of forces having the same maximum but widely different mean values, and from some points of view the mean value may possess more importance than the maximum. This is the chief reason for recording in the inequality tables the average of the 24 hourly departures from the mean for the day. In previous papers I have usually recorded not the average departure but the sum of the 24 hourly differences from the mean. The one result is, of course, immediately derivable from the other.

Table XI. gives particulars of the ratio borne by the average departure from the mean to the inequality range. There are two sets of data as in Table X., the first representing arithmetic means of the ratios calculated for the 12 months of the year individually, the second the corresponding values for the seasonal inequalities. For the sake of comparison, results are given for quiet as well as for disturbed days. In the case of D and V the ratio is decidedly higher for the disturbed than for the quiet days. In other words, a comparison based on the range alone would underestimate the forces producing the diurnal inequality on disturbed days, as compared to the corresponding forces on quiet days. In the case of I and H there is little, if any, difference as regards the ratio between quiet and disturbed days. The differences between the values of the ratio answering to all the disturbed days and to the smaller number of more highly disturbed days are too small to rely on.

Fourier Coefficients.

§ 18. A diurnal inequality may be analysed in a Fourier series, taking either of the equivalent forms

$$a_1 \cos t + b_1 \sin t + a_2 \cos 2t + b_2 \sin 2t + \dots,$$

$$c_1 \sin (t+\alpha_1) + c_2 \sin (2t+\alpha_2) + \dots,$$

where t denotes time reckoned from 0 a.m., 15° being taken as the equivalent of one hour.

The natural order is to calculate the a and b constants from the hourly inequality values, and then derive the c (amplitude) and α (phase angle) constants from the formula

$$\tan \alpha_n = a_n/b_n, \quad c_n = a_n/\sin \alpha_n = b_n/\cos \alpha_n.$$

Physical interest attaches mainly to the c and α constants, and especially to those of the first two terms, which represent the 24-hour and 12-hour "waves."

In the present case there is inevitably an appreciable "accidental" element in the values obtained for individual months, especially in the case of the 8-hour and 6-hour waves, to say nothing, of course, of higher terms which we shall leave out of account. To economise space, the values of the a and b constants are not given here, and values of c and α for individual months are limited to the 24- and 12-hour waves. These are given for H, V, and I in Table XII. Arithmetical means from the 12 monthly values obtained for the c constants, and the values derived for the c and α constants from the yearly and seasonal diurnal inequalities are given in Table XIII., p. 296. This includes the 8- and 6-, as well as the 24- and 12-hour waves. For the c constants the unit is 1γ for H and V, and $1'$ for I.

TABLE XII.—Fourier Coefficients, Amplitudes, and Phase Angles.

	Horizontal force.				Vertical force.				Inclination.			
	c_1.	α_1.	c_2.	α_2.	c_1.	α_1.	c_2.	α_2.	c_1.	α_1.	c_2.	α_2.
		°		°		°		°		°		°
January	3·8	347	4·5	270	11·4	178	3·1	242	0·56	173	0·23	100
February	5·8	198	8·7	286	34·9	198	11·2	262	0·58	198	0·32	129
March	6·2	167	9·6	307	27·8	191	13·6	265	0·43	214	0·44	161
April	16·6	125	15·4	310	26·8	181	12·7	278	0·92	263	0·75	145
May	25·9	144	17·8	312	42·3	194	16·7	294	1·32	282	0·76	143
June.	34·7	142	20·1	318	39·7	191	16·5	296	1·80	294	0·92	149
July.	43·2	155	21·9	314	52·7	190	11·1	273	1·89	308	1·25	144
August	34·2	142	22·1	320	38·0	185	2·0	114	1·68	296	1·51	140
September	14·4	123	14·4	325	28·0	200	10·1	272	1·08	263	0·79	160
October.	11·8	88	5·7	287	15·3	186	6·2	274	0·95	242	0·21	119
November	10·2	51	6·1	291	25·3	202	7·5	277	1·34	216	0·24	131
December	7·3	338	6·8	259	14·1	177	3·6	261	0·87	166	0·35	79

§ 19. Table XII. shows in all three elements a marked and fairly regular rise in the values of both c_1 and c_2 from midwinter to midsummer. There was, it must be remembered, an excess in the mean absolute ranges in the summer months over those in the winter months, especially in the case of V; but the difference between the summer and winter values of c_1 and c_2 is considerably more prominent, especially in H.

This supports the conclusion, suggested by general inspection of the diurnal inequalities, that the nature of the changes of larger feature which take place in the course of a magnetic storm are largely dependent on the season of the year when the storm occurs. Some external influence may, so to speak, pull the trigger, but the nature and amplitude of the disturbance are influenced in an important degree by the local time and by the season of the year.

If "accidental" influences had been dominant in the diurnal inequalities obtained for individual months, the amplitudes and phase angles in Table XII. would have varied erratically from month to month. It cannot, of course, be claimed that there are no erratic values. Those for c_2 and α_2 in August for V are unquestionably outstanding, but, on the whole, the smoothness of the monthly values, especially in summer when the amplitude is large, is not a little remarkable.

§ 20. Values of the a, b, c, and α constants were calculated for the seasonal inequalities derived from the 10 days a month of largest absolute range and from the days of the four sunspot maximum years. Of these the c and α constants are given in Table XIII., alongside the constants derived from all the disturbed days. Arithmetic mean values from the 12 months, for all disturbed days, are also given for the c constants, as some authorities prefer them to the values derived from the mean diurnal inequality for the year. Unless the phase angle is invariable throughout the year—a most unlikely contingency, even if solar time were used—the arithmetic mean value necessarily exceeds that derived from the mean diurnal inequality. In cases

TABLE XIII.—Fourier Coefficients, Amplitudes, and Phase Angles.

		c_1.	α_1.	c_2.	α_2.	c_3.	α_3.	c_4.	α_4.
			°		°		°		°
H	Arithmetic mean	17·9	—	12·8	—	4·1	—	2·6	—
	Year, all days.	14·4	137	12·3	309	2·5	163	1·6	42
	Winter, all days	2·9	12	6·4	279	2·4	155	0·8	15
	„ 40 days	2·7	283	8·5	286	2·5	139	0·8	39
	Equinox, all days . . .	11·3	121	11·1	311	3·4	160	2·4	56
	„ 40 days . . .	12·5	134	13·5	315	4·2	170	2·3	54
	Summer, all days . . .	34·3	146	20·4	316	1·8	182	1·7	35
V	Arithmetic mean	29·7	—	9·5	—	4·1	—	2·1	—
	Year, all days.	29·5	191	9·0	277	3·6	62	0·8	264
	„ sunspot maximum .	41·5	196	11·4	271	4·1	56	1·1	276
	Winter, all days	21·1	193	6·2	264	2·6	125	1·6	198
	„ 40 days	31·1	199	9·9	257	4·3	127	2·9	192
	„ sunspot maximum	34·7	201	11·5	249	3·9	134	2·7	176
	Equinox, all days . . .	24·3	190	10·6	272	3·4	42	1·1	351
	„ 40 days . . .	33·5	197	14·1	276	3·6	35	0·9	319
	„ sunspot maximum	33·2	195	13·4	266	2·2	22	1·8	323
	Summer, all days . . .	43·1	190	10·4	289	6·5	52	1·7	274
	„ sunspot maximum	56·9	194	11·6	299	10·2	41	2·7	306
I	Arithmetic mean	1·12	—	0·65	—	0·28	—	0·19	—
	Year, all days.	0·81	263	0·62	141	0·21	12	0·13	228
	Winter, all days	0·78	193	0·26	108	0·11	355	0·09	196
	Equinox, all days . . .	0·81	251	0·53	151	0·27	357	0·15	242
	Summer, all days . . .	1·65	296	1·11	143	0·27	33	0·15	231

where the phase angle varies largely with the season of the year, the arithmetic mean value gives the better idea of the mean amplitude of the forces to which the Fourier "wave" in question is due. A very close accordance between the two sets of values indicates that the phase angle is nearly constant throughout the year, with the possible exception of months in which the "wave" in question is relatively small. Another pretty safe inference in such a case is that the monthly values have not suffered seriously from defects in the observational data.

With the exception of c_1 in H and I, the arithmetic mean and the yearly inequality values for c_1 and c_2 in Table XIII. differ but little. There seems to be a considerable seasonal variation in α_1 in H and I, but c_1 is so small in some winter months that more than usual uncertainty attaches to the values for α_1 at this season. The difference between the arithmetic mean and yearly inequality values for c_3 and c_4 is more conspicuous, and probably reflects uncertainty in the monthly data rather than true seasonal variability in the phase angles. There is, in fact, indication of but a small seasonal variation in the values obtained for α_3 and α_4 in the case of H.*

The values obtained for c_1 and c_2 in V from the smaller number of more highly disturbed days are much larger than those derived from all the disturbed days. There is, however, no such enhancement in the winter value of c_1 in H, and the results for c_3 and c_4 are in this respect somewhat conflicting in all the elements. The 8- and 6-hour waves may in reality be but little influenced by disturbance. Another possible explanation, however, is that the greater irregularities existing in the data from the smaller number of days tended to neutralise the increased amplitude of disturbance. It is the latter explanation presumably that applies to the winter value of c_1 in H.

§ 21. Table XIV. gives the ratios borne by the amplitudes of the 12-, 8-, and 6-hour waves to that of the 24-hour wave. Older results for D are included. In D and I the disturbed day results are based on the total number of disturbed days. The relative importance of the shorter period waves tends to diminish on disturbed days, especially in V. The 12-hour term in H presents, however, rather a marked exception to this rule, and the same is true to a lesser extent in I.

The great reduction in the relative importance of the shorter period waves in V arises from the enormous influence of disturbance in increasing the amplitude of the 24-hour wave in that element. If we calculate the ratio which the amplitude of the 24-hour wave from all disturbed days in the mean diurnal inequality for the year bears to the corresponding amplitude from quiet days, we find it to be 2·18 in D, 1·59 in I, and 1·52 in H, but 5·25 in V.

Large as is the value of c_1 in V derived from all the disturbed days, it is exceeded by 40 per cent. by the corresponding value derived from the sunspot maximum years.

It would be interesting to know the corresponding sunspot effect on quiet days, but this unfortunately was not determined in (A).

TABLE XIV.—Ratios Borne by the Amplitudes of the 12-, 8-, and 6-hour Fourier Waves to that of the 24-hour Wave.

		c_2/c_1.				c_3/c_1.				c_4/c_1.			
		Winter.	Equinox.	Summer.	Year.	Winter.	Equinox.	Summer.	Year.	Winter.	Equinox.	Summer.	Year.
D	Quiet days	0·65	0·81	0·78	0·77	0·36	0·47	0·32	0·38	0·18	0·19	0·04	0·12
	All disturbed days	0·31	0·44	0·31	0·35	0·10	0·11	0·25	0·16	0·10	0·20	0·12	0·10
I	Quiet days	0·92	0·48	0·40	0·49	0·43	0·35	0·17	0·30	0·32	0·21	0·09	0·17
	All disturbed days	0·34	0·66	0·67	0·76	0·14	0·34	0·17	0·26	0·12	0·19	0·09	0·15
H	Quiet days	1·07	0·54	0·42	0·54	0·50	0·30	0·16	0·25	0·31	0·17	0·06	0·14
	All disturbed days	2·17	0·98	0·60	0·86	0·82	0·30	0·05	0·17	0·26	0·21	0·05	0·11
	10 days a month	3·13	1·08	—	—	0·91	0·34	—	—	0·30	0·19	—	—
V	Quiet days	0·68	0·76	0·75	0·75	0·35	0·41	0·26	0·33	0·17	0·15	0·06	0·11
	All disturbed days	0·30	0·44	0·24	0·30	0·12	0·14	0·15	0·12	0·07	0·05	0·04	0·03
	10 days a month	0·32	0·42	—	—	0·14	0·11	—	—	0·09	0·03	—	—
	Sunspot maximum years	0·33	0·40	0·21	0·28	0·11	0·07	0·18	0·10	0·08	0·05	0·05	0·03

§ 22. Table XV. supplies the means of comparing the Fourier "wave" phase angles derived from all the disturbed days with the corresponding angles for quiet days. The quantity tabulated is the algebraic excess of the disturbed over the quiet day phase angle. An increase of phase angle signifies an earlier occurrence in the maxima and minima of the corresponding wave, an hour's advance in time answering to 15° in α_1, 30° in α_2, 45° in α_3, and 60° in α_4. Results are given for the three seasonal diurnal inequalities as well as for the mean diurnal inequality for the year. Large

TABLE XV.—Algebraic Excess of Disturbed Days' Phase Angle over Quiet Days' Phase Angle.

Element.	α_1.				α_2.				α_3.				α_4.			
	Year.	Winter.	Equinox.	Summer.	Year.	Winter.	Equinox.	Summer.	Year.	Winter.	Equinox.	Summer.	Year.	Winter.	Equinox.	Summer.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
D	+34·9	+30·8	+37·8	+28·5	−32·6	−43·0	−11·2	−48·0	−37	−55	−45	−31	+49	+11	+86	+97
I	−31·7	−56·8	−36·7	−16·1	+ 9·1	+16·7	+16·2	−11·9	+ 1	+12	− 6	− 5	+18	+ 3	+36	− 5
H	+20·4	−70·4?	+11·6	+16·4	+ 6·7	+ 1·9	+ 8·5	+ 0·3	−10	+ 2	− 7	−16	+25	+11	+41	− 4
V	+66·5	+39·6	+73·0	+68·5	+ 1·5	−36·0	+ 0·3	+17·5	−38	+18	−55	−47	−21	−81	+63	−10

differences between the seasonal results—for example, the difference between the winter and other seasonal results under α_1 in H—are to be regarded with some suspicion. But there are several reasons for not expecting a very close agreement. If we compare the phase angles in Table XIII., which are derived from all the disturbed days, with those derived from the smaller number of more highly disturbed days, or with those derived from sunspot maximum years, we find a difference which is generally in the direction suggesting that the departure from the value of the phase angle characteristic of quiet days is greater the more disturbed the conditions. For instance, α_1 in V is at all seasons much larger for disturbed than for quiet days, and in Table XIII. the value of α_1 derived from the whole body of disturbed days is invariably the least. Thus, even if there were no true seasonal variation in the difference between the phase angles characteristic of quiet days and of days possessing a standard amount of disturbance, we should expect some seasonal variation to appear in Table XV. through seasonal fluctuation in the amount of disturbance. Finally, quiet day phase angles do show more or less seasonal variation, and there is no obvious reason why such seasonal variation should be identical for days of standard disturbance.

Comment was passed in (C) on the great increase in α_1 in D through disturbance, representing on the average an advance of over two hours in time. The corresponding change in V is, however, an advance of over four hours. The corresponding change in I is retrograde and roughly equal to that in D. The winter results for α_1 in H had better be regarded as unreliable. At the other seasons α_1 in H shows an advance of time, though considerably less than in D. The influence of disturbance on α_2, α_3, and α_4, when regarded from the point of view of the equivalent in time, is in general relatively small, and is decidedly less in I, H, and V than in D. The differences between the disturbed and quiet day values of α_3 and α_4 in the three former elements are in fact generally less than one would have anticipated even on the hypothesis that the phase angles are really unaffected by disturbance.

Force Vector of Diurnal Inequality.

§ 23. The departure of the magnetic elements from their mean value for the day may be regarded as due to a disturbing force of constantly varying intensity and direction. Table XVI. gives particulars of the direction and intensity of this force at each hour of the day for the year as a whole. Analogous results could, of course, be derived from the diurnal inequalities for individual months or seasons of the year.

ϕ denotes the angle which the vertical plane containing the resultant force makes with the vertical plane through the geographical meridian, measured from north to east. θ denotes the inclination of the resultant force or vector to the vertical, measured from the nadir. The force on the north pole of a magnet is thus directed below or above the horizontal plane according as θ is less or greater than 90°. R denotes the intensity of the resultant force or vector, ρ its component in the

TABLE XVI.—Co-ordinates of Diurnal Inequality Force Vector.

Hour.	ϕ.			θ.			ρ (unit 1γ).			R (unit 1γ).		
	q.	d.	$d-q$.	q.	d.	$d-q$.	q.	d.	$d-q$.	q.	d.	$d-q$.
	°	°	°	°	°	°						
1 a.m.	31	72	84	79	124	131	6·3	25·8	21·4	6·4	31·2	28·5
2 ,,	37	66	76	84	134	143	**6·0**	21·6	16·6	**6·0**	30·2	27·3
3 ,,	38	56	64	87	138	149	6·5	19·8	13·7	6·5	29·7	26·4
4 ,,	43	49	59	86	156	172	7·9	11·5	**3·7**	7·9	27·8	26·1
5 ,,	50	6	266	86	166	164	10·2	6·0	7·2	10·2	25·3	26·3
6 ,,	62	38	249	88	172	157	12·5	**3·1**	9·7	12·5	23·1	25·3
7 ,,	76	138	236	89	164	144	15·0	5·1	13·4	15·0	**18·5**	22·5
8 ,,	95	146	220	93	136	135	17·6	14·8	14·3	17·6	21·2	20·1
9 ,,	117	162	213	102	120	120	18·9	24·3	17·2	19·3	28·0	19·9
10 ,,	151	180	213	114	114	107	18·4	29·9	16·7	20·1	32·8	17·5
11 ,,	190	199	211	120	108	89	19·7	34·1	15·0	22·7	35·8	15·0
12 ,,	222	221	221	115	101	73	24·2	38·0	13·8	26·8	38·7	**14·4**
1 p.m.	238	239	242	108	87	50	**26·6**	**38·5**	12·0	**27·9**	38·6	15·7
2 ,,	248	258	274	98	70	42	23·4	38·0	15·5	23·6	40·4	23·1
3 ,,	257	259	282	85	50	32	16·7	32·6	16·0	16·8	42·3	30·0
4 ,,	269	284	294	64	33	26	9·7	25·2	16·1	10·8	**45·7**	37·1
5 ,,	303	306	307	45	27	27	6·2	23·3	17·1	8·9	44·8	**37·7**
6 ,,	335	337	338	47	26	20	7·0	18·4	11·4	9·5	42·4	33·7
7 ,,	346	5	17	53	32	26	8·3	20·3	12·6	10·4	38·2	29·1
8 ,,	356	42	62	56	44	46	8·0	21·3	17·2	9·7	30·6	23·6
9 ,,	4	59	76	59	61	67	7·7	24·6	21·1	9·0	28·1	22·9
10 ,,	13	68	86	63	85	96	7·0	20·7	17·5	7·9	20·7	17·6
11 ,,	20	63	76	70	98	106	7·0	26·4	**21·8**	7·5	26·6	22·7
12 ,,	30	65	80	75	121	132	6·9	20·9	15·8	7·1	24·5	21·5

horizontal plane. The ρ, ϕ curve in the horizontal plane is that usually termed the "vector diagram."

If ΔN, ΔE (or $-\Delta W$), and ΔV denote departures of the northerly, easterly, and vertical components of force from their mean values,

$$\phi = \tan^{-1}(\Delta E/\Delta N), \qquad \theta = \tan^{-1}(\rho/\Delta V),$$

$$\rho = \sqrt{(\Delta N^2+\Delta E^2)} = \Delta N/\cos\phi = \Delta E/\sin\phi,$$

$$R = \rho/\sin\theta = \Delta V/\cos\theta.$$

§ 24. The three headings q, d, and $d-q$ in Table XVI. denote respectively results derived from quiet days, results derived from the 209 disturbed days, and results derived from the difference inequalities, which are obtained by subtracting quiet day from disturbed day hourly values. If the causes operative on quiet days continue to exert the same influence on disturbed days, then the results in the $d-q$ columns represent the disturbance element pure and simple. The calculations were carried at least one decimal place further than appears in the table.

A drawing of the ρ, ϕ curve on quiet days will be found in (A) (fig. 11, p. 377). In this case there is no retrograde motion, ϕ continually increasing throughout the

24 hours. 4° at 9 p.m. is, of course, 364° relatively to 356° at 8 p.m. The curve, as viewed by an observer looking down on the horizontal plane, is described in the direction of the hands of a watch. The motion is also in the direction of the hands of a watch on disturbed days throughout the greater part of the day, but it is distinctly retrograde for some hours after midnight. In the $d-q$ column there is an apparent discontinuity in the value of ϕ between 4 and 5 a.m. This really means that ρ vanishes and the curve crosses the geographical meridian.

The disturbed-day vector is in advance of the quiet-day vector at most hours of the day. They are, however, practically coincident at noon and remain close together until 6 p.m.

§ 25. On quiet days θ increases from a minimum at 5 p.m. until at least 2 or 3 a.m., and then remains nearly constant until 7 a.m. The rise to the maximum at 11 a.m. and subsequent fall are rapid. The vector is directed above ground only from 8 a.m. to 2 p.m. The total range in θ is 75°, from 45° below to 30° above the horizon.

On disturbed days the range of θ is much wider, being from 64° below to 82° above the horizon, or in all 146°; the rise from 6 p.m. to 6 a.m., and the fall from 6 a.m. to 6 p.m., are continuous.

The mean values of θ for the q, d, and $d-q$ columns respectively are 82°, 99°, and 94°, so that the mean position is about as much above the horizon on disturbed days as it is below it on quiet days.

§ 26. The absolutely largest and smallest values of ρ and R in Table XVI. are in heavy type. The disturbed day value of ρ is larger than the quiet day value except from 5 a.m. to 8 a.m. On neither type of day is there much variation of ρ between 5 p.m. and 3 a.m. On quiet days, owing to the comparatively small influence of V, the value of R is usually but little larger than that of ρ; both quantities have a maximum at 1 p.m. and a minimum at 2 a.m. On disturbed days R is usually considerably larger than ρ, and has its maximum 3 hours and its minimum 1 hour later. The disturbed day value of R is always larger than the quiet-day value.

§ 27. If we regard 6 a.m. to 6 p.m. as "day," and ascribe half the sum of the values at these two hours to "day" and half to "night," we obtain the following mean values for ρ and R, employing q, d, and $d-q$ in the same sense as before :—

TABLE XVII.—Comparison of Day and Night Values of Diurnal Inequality Vector.

Mean values.	ρ (unit 1γ).			R (unit 1γ).		
	q.	d.	$d-q$.	q.	d.	$d-q$.
For 24 hours	12·4	22·7	14·9	13·3	31·9	24·3
For "day"	17·2	26·2	14·8	18·4	35·0	23·5
For "night"	7·6	19·1	14·9	8·3	28·8	25·1
(Mean "night"/mean "day") . . .	0·44	0·73	1·01	0·45	0·82	1·07

The great reduction in the difference between day and night as we pass from quiet to disturbed days seems noteworthy. The accordance between day and night in the $d-q$ columns is remarkable.

§ 28. When dealing with the phenomena at a single station, the vertical is probably the most convenient initial line. If, however, we wished to compare results at stations in different latitudes, it would probably be found more convenient to define the resultant force vector by the intensity R—which is, of course, the same for all co-ordinate systems—the angle ψ which R makes with the earth's axis, and the angle χ which a plane parallel to the earth's axis and containing R makes with the meridian plane.

If λ denote the latitude of the station, and θ and ϕ have their previous meanings, and if P and Q denote the components of R in the meridian plane, respectively parallel and perpendicular to the earth's axis, then

$$P = R\,(\sin\lambda\,\cos\theta - \cos\lambda\,\sin\theta\,\cos\phi),$$

$$Q = R\,(\cos\lambda\,\cos\theta + \sin\lambda\,\sin\theta\,\cos\phi).$$

The positive direction of P is here taken from the North to the South Pole, and the positive direction of Q towards the earth's axis. The third rectangular co-ordinate of R is ΔE (or $-\Delta W$), *i.e.*, $R \sin\theta \sin\phi$.

The relations connecting ψ and χ with θ and ϕ are

$$\psi = \cos^{-1}(\sin\lambda\,\cos\theta - \cos\lambda\,\sin\theta\,\cos\phi),$$

$$\chi = \cot^{-1}(\cos\lambda\,\cot\theta\,\operatorname{cosec}\phi + \sin\lambda\,\cot\phi).$$

At Kew, where $\lambda = 51^\circ\,28'\,6''$, these become

$$\psi = \cos^{-1}(\cdot782\,\cos\theta - \cdot623\,\sin\theta\,\cos\phi),$$

$$\chi = \cot^{-1}(\cdot623\,\cot\theta\,\operatorname{cosec}\phi + \cdot782\,\cot\phi).$$

Comparison of Diurnal Inequalities at Different Stations.

§ 29. The results arrived at in this and my previous papers suggest a number of considerations relating to the tabulation and interpretation of magnetic curves. Some stations in deriving diurnal inequalities limit themselves to a selection of the quieter days. There may be only five of these a month, as at Kew and Falmouth, or 10, as at the stations of the U.S. Coast and Geodetic Survey. Other stations derive diurnal inequalities from all days, excepting as a rule those specially disturbed. A few stations, *e.g.*, Greenwich and Pawlowsk, publish regularly inequalities of two kinds, viz., from quiet days alone—the Pawlowsk quiet days are usually only two or three a month—and from all but highly disturbed days. At some stations the curves, even those from quiet days, are systematically smoothed; at others mean hourly values are obtained instrumentally, and there are some, I believe, where smoothing of any kind is exceptional or non-existent. The diurnal inequality data being published at present are thus not comparable, while it is desirable that really parallel data should be available for the use of those who wish to study the general laws or to find a physical explanation of the phenomena.

The best means of securing more homogeneity may depend on the true nature of the difference between days of different types. Limiting our view for a moment to a particular season of a particular year, our researches show that there is no such thing as a single unique type of diurnal inequality. The simplest case now conceivable is that there are two extreme types, the one asymptotic for quiet days, the other for highly disturbed days. As regards the second hypothetical type, there are several conceivable possibilities. The inequality seen on quiet days may exist equally on disturbed days, being simply supplemented by an inequality not present on ideally quiet days. On the other hand, the inequality seen on quiet days may be replaced to a greater or less extent by a different type of inequality on disturbed days, being wholly unrepresented on the ideal infinitely disturbed day. In either case there is the possibility that the difference in the phenomena on quiet and disturbed days is due not to any real difference in the forces to which the diurnal inequality is ultimately due, but to a difference of condition in the medium where the forces act.

If the forces which produce the diurnal inequality on quiet days act and produce the same effect on disturbed days, then the type to which highly disturbed days show an asymptotic approach might be regarded as resulting from the direct superposition of the ideally quiet day inequality and of an inequality which represents disturbance pure and simple. The ideally quiet day inequality would then enter into the inequality derived from any group of disturbed days, and by subtracting the ideal quiet day inequality we should obtain an inequality representing disturbance alone. It would not be really necessary to know the ideal quiet day inequality, as it would be eliminated by subtracting from one another the inequalities derived from any two groups of days which represent different intensities of disturbance. If we then had three groups of days A, B, and C, of which A included the most quiet and C the most disturbed days, the inequalities B−A, C−B, and C−A would be of the same type, but would differ in range. The range in such a difference inequality would, in short, be a measure of the difference in the scale of disturbance.

§ 30. A serious complication in the way of direct appeal to observation arises from the sunspot influence. As was shown fully in (A), there is a difference between the diurnal inequalities derived from quiet days in years of sunspot maximum and minimum, which is not represented by mere change of amplitude. This may mean that so-called quiet days are not ideally quiet, and that in years of many and in years of few sunspots they are unequally affected by the residual disturbance attaching to them. This, however, is pure conjecture. The disturbed days dealt with in (C) and the present paper are taken from years of very varying sunspot frequency.

They were selected in a somewhat arbitrary way, and many more were taken from some years (*e.g.*, 1892 and 1896) than from others (1890, 1893, and 1900). They are also in all probability not equally representative of the different seasons of the year. It is thus impossible to say what the exact significance is of the difference in type—which is not very striking—between the disturbed ∽ quiet and the ordinary ∽ quiet day inequalities of declination shown in (C), fig. 3.

§ 31. The V difference curves in fig. 2 of the present paper are of a very suggestive character. Anyone, I think, looking at the three sets of V curves in this figure, if knowing only that two were fundamental and one derived, would naturally select the quiet day and the difference curves as the fundamental ones. The quiet day curve has for its essential features the deep depression about noon, the late forenoon fall, the early afternoon recovery. The difference curve has for its essential features the early morning minimum, the smooth rise to the afternoon maximum, and the smooth evening fall. The disturbed day curve seems to represent a struggle, only partially successful, to throw off the midday depression and to approach an ideal represented by the difference curve.

The difference in type between disturbed and quiet day curves seems much less in H and N than in V or W. There may conceivably be directions in space at each station, the components along which of the diurnal force system are unaffected in type by disturbance. If such directions exist, their discovery and utilisation might lead to valuable results.

§ 32. A conclusion to which my investigations point is that, if circumstances allow, much knowledge might be gained by dividing the days at each station into, say, three not overlapping groups, one representing quiet days, one highly disturbed days, and the third intermediate days, diurnal inequalities being formed from these groups separately. With our present knowledge there is no means of securing that the choice of the different groups will lead to equality in the degree of disturbance in the same month at different stations, or at the same station in different months. Any very complicated system of selection would be pretty certain to break down. The best plan, until further knowledge is gained, would probably be to specify a given number of days for each of the extreme groups. Judging by the international results, already referred to, the choice in the case of quiet days would lie between 5 and 10 days a month. In most months there are at least 10 moderately quiet days at stations in temperate latitudes, and if the first group were limited to 5 days it would mean including in Group B (intermediate days) a number of days more appropriate to Group A. On the other hand, it is often difficult in high latitudes to get even 5 moderately quiet days a month, and to get 10 is difficult in some months even in temperate latitudes. Then some stations may be willing to tabulate 5 but not as many as 10 days a month, and it would be a pity to lose their co-operation entirely.

For the Group C of highly disturbed days, 5 promises to be the best number.

This would make the standard of disturbance lower than in the present paper, and lower than the average interpretation of the international scale "2." If, however, only 1 or 2 days were taken from each month, a very large number of years' data would have to accumulate before anything like a reasonably smooth diurnal inequality could be deduced for individual months of the year, and this even when years of many and years of few sunspots were grouped indiscriminately. Taking even as many as 5 days a month, it would probably be necessary to combine all the months of one season to derive moderately smooth inequalities from the data of one year. But the accumulated data of an 11-year period would probably suffice to give tolerably smooth results for individual months, treating years of many and of few sunspots separately.

In some months, no doubt, to get as many as 5 days, one would have to include days which did not attain to any very high standard of disturbance, but it must be remembered that many of the days which are classed "1" on the international scale are so nearly of standard "2" that considerable divergence of view as to their classification may exist even at stations so near together as Greenwich and Kew.

§ 33. In connection with highly disturbed days, there is the difficulty that disturbance occasionally leads to loss of trace, and at some stations this is rather a frequent occurrence. To show clearly the smaller movements on quiet and ordinary days a high degree of sensitiveness is desirable, and this is hardly compatible with including in the width of the photographic sheet the range of the largest disturbances. To obviate this, the magnets in some magnetographs carry more than one mirror, so that when the light from one gets beyond the edge of the photographic sheet, light from another may remain on. This, however, introduces some complication, and at times of large rapidly oscillatory disturbance confusion may be introduced. A better plan, if financial conditions allow, is to have two magnetographs, one sufficiently insensitive to preclude loss of trace under any contingency that can be reasonably anticipated.

§ 34. In the case of highly disturbed curves, I do not think that ordinary smoothing by hand is at all likely to prove satisfactory. An hourly mean derived from the area between the curve and base line and the limiting ordinates may be more satisfactory, but during rapidly oscillating disturbances this would be a difficult method to apply. In the case of the "Discovery" Antarctic curves, ordinates were measured at the hour and at 20 minutes before and after, and the mean of the three accepted. With the aid of a suitable scale this proved a comparatively short process, and it unquestionably produced a remarkable smoothing of the results. By adopting some similar method it would be possible to obtain fairly smooth inequalities from a comparatively small number of disturbed days. I hope the opportunity may be found at Kew to ascertain the effect, if any, of the smoothing process adopted there in the case of ordinary day curves.

Much of the arithmetical work on which the paper is based was done by various members of the staff of the Observatory Department—especially Messrs. B. FRANCIS and W. R. CORRIN, junior—to whose care and accuracy I am much indebted. The calculations were all checked by myself. The cost of tabulating the curves was partly defrayed out of a grant which I received two years ago from the Government Grant Committee.